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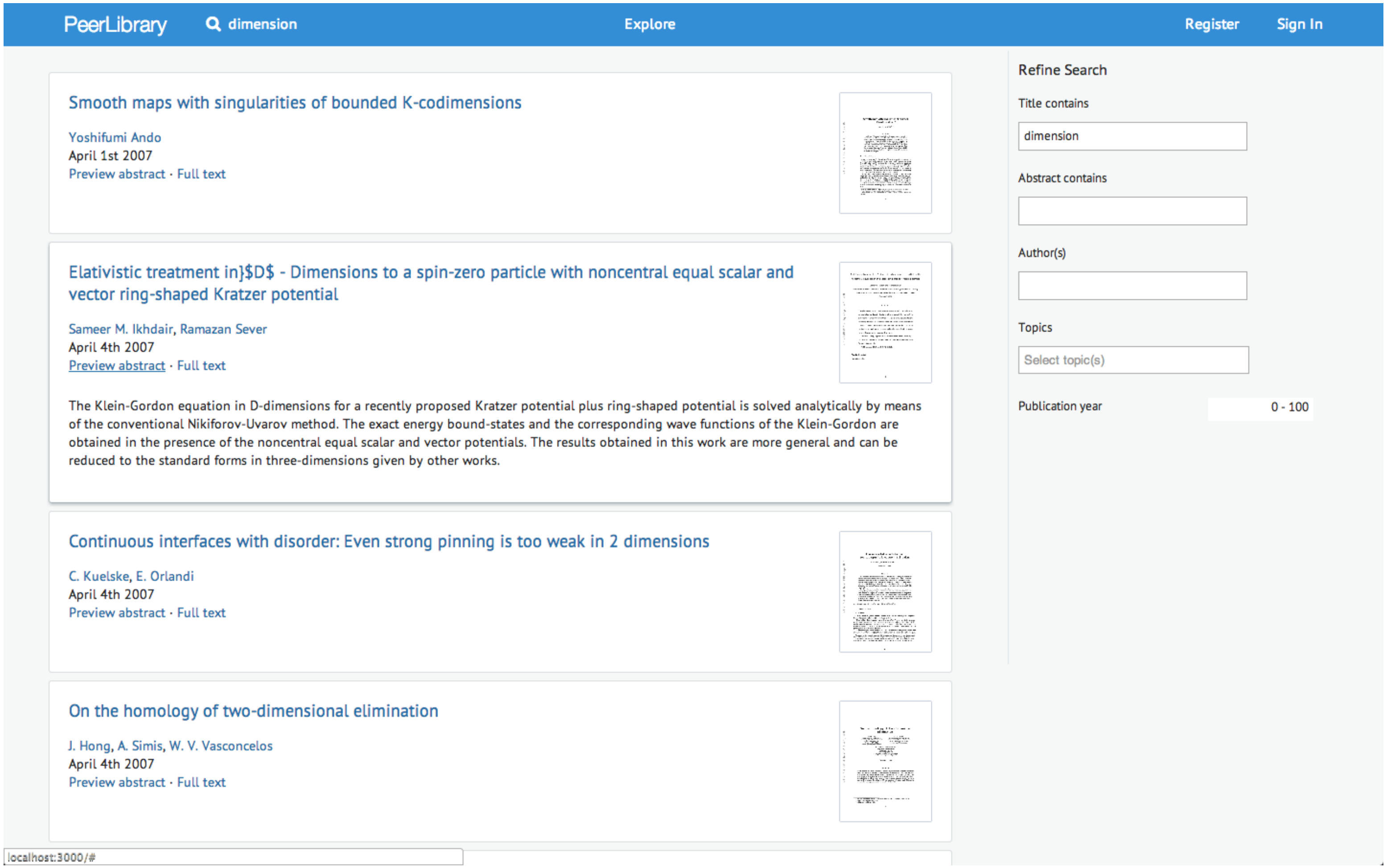

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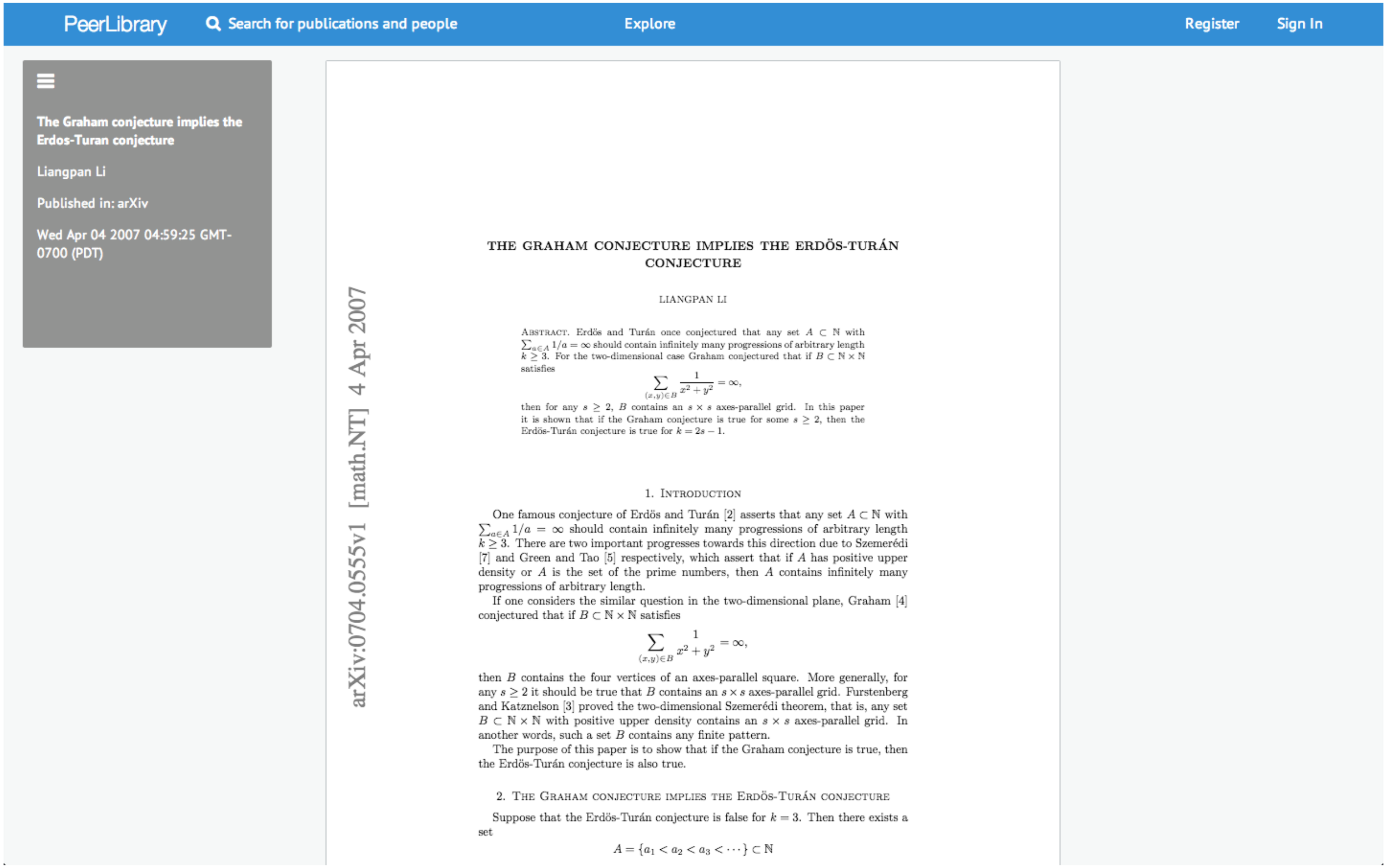

THE GRAHAM CONJECTURE IMPLIES THE ERDÖS-TURÁN CONJECTURE

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ABSTRACT. Erdös and Turán once conjectured that any set $A \subset \mathbb{N}$ with $\sum_{a\in A} 1/a = \infty$ should contain infinitely many progressions of arbitrary length $k \geq 3$. For the two-dimensional case Graham conjectured that if $B \subset \mathbb{N}\times\mathbb{N}$ satisfies

$$\sum_{(x,y)\in B} \frac{1}{x^2+y^2} = \infty,$$

then for any $s \geq 2$, B contains an $s \times s$ axes-parallel grid. In this paper it is shown that if the Graham conjecture is true for some $s \geq 2$, then the Erdös-Turán conjecture is true for $k = 2s - 1$.

1. INTRODUCTION

One famous conjecture of Erdös and Turán [2] asserts that any set $A \subset \mathbb{N}$ with $\sum_{a\in A} 1/a = \infty$ should contain infinitely many progressions of arbitrary length $k \geq 3$. There are two important progresses towards this direction due to Szemerédi [7] and Green and Tao [5] respectively, which assert that if A has positive upper density or A is the set of the prime numbers, then A contains infinitely many progressions of arbitrary length.

If one considers the similar question in the two-dimensional plane, Graham [4] conjectured that if $B \subset \mathbb{N}\times\mathbb{N}$ satisfies

$$\sum_{(x,y)\in B} \frac{1}{x^2+y^2} = \infty,$$

then B contains the four vertices of an axes-parallel square. More generally, for any $s \geq 2$ it should be true that B contains an $s \times s$ axes-parallel grid. Furstenberg and Katznelson [3] proved the two-dimensional Szemerédi theorem, that is, any set $B \subset \mathbb{N}\times\mathbb{N}$ with positive upper density contains an $s \times s$ axes-parallel grid. In another words, such a set B contains any finite pattern.

The purpose of this paper is to show that if the Graham conjecture is true, then the Erdös-Turán conjecture is also true.

2. THE GRAHAM CONJECTURE IMPLIES THE ERDÖS-TURÁN CONJECTURE

Suppose that the Erdös-Turán conjecture is false for $k = 3$. Then there exists a set

$$A = \{a_1 < a_2 < a_3 < \cdots\} \subset \mathbb{N}$$

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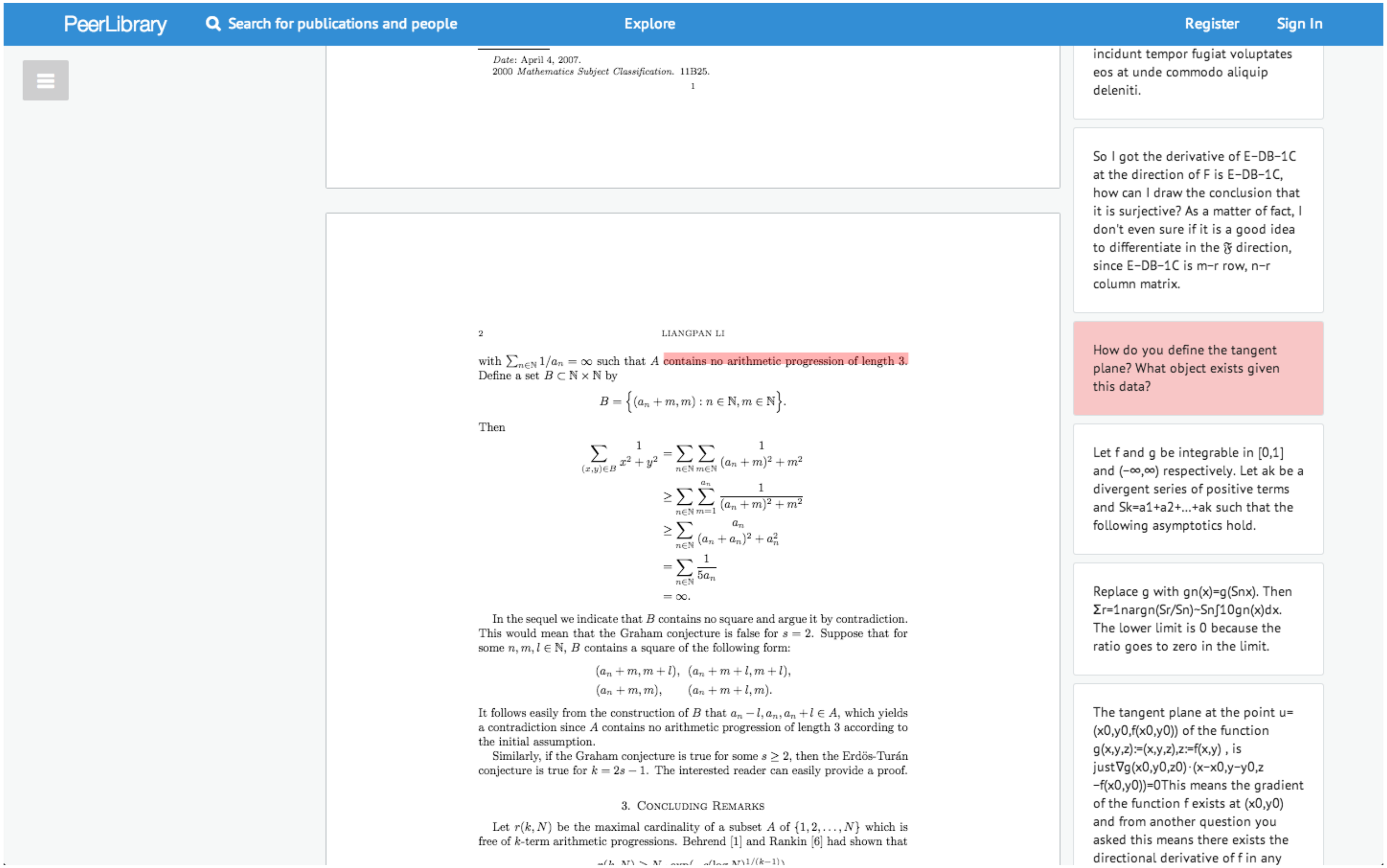

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